Sets

- A set is a well-defined collection of objects.
  Example: The collection of all rational numbers less than 10 is a set whereas the collection of all the brilliant students in a class is not. This is because the collection of all the brilliant students in a class is not well defined. The criterion for determining a brilliant student may vary from person to person.
- The following points are noted while writing a set.
  - Sets are usually denoted by capital letters A, B, S, etc.
  - The elements of a set are usually denoted by small letters a, b, t, u, etc.
  - Symbols for some sets particularly used in mathematics:
    N, Z, Q, R, $\mathbf{Z^+}$, $\mathbf{Q^+}$, $\mathbf{R^+}$ denote the set of all natural numbers, the set of all integers, the set of all rational numbers, the set of all real numbers, the set of all positive integers, the set of all positive rational numbers and the set of all positive real numbers respectively.
- If x is an element of a set S, then we say that "x belongs to S". Mathematically, we write it as $x \in S$. If y is not an element of a set S, then we say that "y does not belong to S". Mathematically, we write it as $y \notin S$.
- There are two methods of representing a set.
  - **Roster or tabular form:** In this form, all the elements of a set are listed, separated by commas and enclosed within braces $\{\ \}$. In this form, the order in which the elements are listed is immaterial, and the elements are not repeated.
    Example: The set of letters forming the word 'TEST' is $\{T, E, S\}$.
  - **Set-builder form:** In this form all the elements of a set possess a single common property which is not possessed by any element outside the set.
    Example: The set $\{2, -2\}$ can be written in the set-builder form as $\{x : x$ is an integer and $x^2 - 4 = 0\}$.
- A set which does not contain any element is called an empty set or a null set or a void set. It is denoted by the symbol ϕ or $\{\ \}$.
  Example: The set $\{x : x \in \mathbf{N}, x$ is an even number and $8 < x < 10\}$ is an empty set.
- A set which is empty or consists of a definite number of elements is called finite; otherwise, the set is called infinite.
  Example: The set $\{x : x \in \mathbf{N}$ and x is a square number$\}$ is an infinite set. The set $\{x : x \in \mathbf{Z}$ and $x^2 - 2x - 3 = 0\}$ is a finite set as it is equal to $\{-1, 3\}$.
- All infinite sets cannot be described in the roster form. For example, the set of rational numbers cannot be described in this form. This is because the elements of this set do not follow any particular pattern.
- Two sets A and B are said to be equal if they have exactly the same elements and we write $A = B$; otherwise, the sets are said to be unequal and we write $A \neq B$.

Example: The sets $A=\{x : x \in \mathbf{N} \text{ and } (x-1)(x+4)=0\}$ and B = {–4, 1} are equal sets.

- A set does not change if one or more elements of the set are repeated.
  Example: The sets A = {E, L, E, M, E, N, T, S} and B = {E, L, M, N, T, S} are equal since each element of A is in B, and vice-versa.
- A set A is said to be a subset of a set B if every element of A is also an element of B and we write $A \subset B$.
  In other words, $A \subset B$ if $a \in A \Rightarrow a \in B$.
  Example: If A = {1, 5} and B = {1, 3, 5, 7}, then every element of set A (i.e., 1 and 5) is an element of set B. In such a case, we write $A \subset B$.
- $A \subset B$ and $B \subset A \Leftrightarrow A = B$
- An empty set is a subset of every set.
- Every set is a subset of itself.
- If $A \subset B$ and $A \neq B$, then A is called a proper subset of B, and B is called a superset of A.
  Note: $\mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q} \subset \mathbf{R}$
- If a set has only one element, then it is called a singleton set.
  Example: $A = \{-17\}$ is a singleton set.
- Intervals as subsets of R:
  Let a, $b \in$ R and $a < b$. Then,
  - $\{y : a < y < b\}$ is called an open interval and is denoted by (a, b). In the open interval (a, b), all the points between a and b belong to the open interval (a, b), but a, b themselves do not belong to this interval.
  - $\{y : a \le y \le b\}$ is called a closed interval and is denoted by $[a, b]$. In this interval, all the points between a and b as well as the points a and b are included.
  - $[a,\ b) = \{y : a \le y < b\}$ is an open interval from a to b, including a, but excluding b.
  - $(a,\ b] = \{y : a < y \le b\}$ is an open interval from a to b, including b, but excluding a.
- The collection of all subsets of a set A is called the power set of A. It is denoted by P (A).
  Example: For the set $A = \{a, b, c\}$, then $P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$
- If A is a set with $n(A) = m$, then $n[P(A)] = 2^m$.
- Most of the relationships between sets can be represented by means of diagrams known as Venn diagrams.
- A universal set is the super set of all sets under consideration and is denoted by U.
  Example: If we consider the sets A, B and C as the cricketers of India, Australia and England respectively, then we can say that the universal set (U) of these sets contains all the cricketers of the world.

- The union of two sets A and B is the set which contains all those elements which are only in A, only in B and in both A and B, and this set is denoted by '$A \cup B$'.
  $A \cup B = \{x : x \in A \text{ or } x \in B\}$

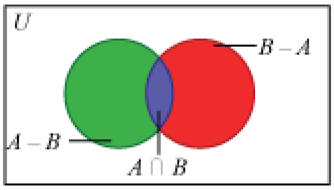
  Example: If $A = \{a, 1, x, p\}$ and $B = \{p, q, 2, x\}$, then $A \cup B = \{a, p, q, x, 1, 2\}$. Here, a and 1 are contained only in A; q and 2 are contained only in B; and p and x are contained in both A and B.
- Properties of the operation of union :
  - $A \cup B = B \cup A$ (Commutative Law)
  - $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative Law)
  - $A \cup \phi = A$ (ϕ is the identity of $\cup$)
  - $A \cup A = A$ (Idempotent Law)
  - $U \cup A = U$ (Law of U)
- The intersection of two sets A and B is the set of all those elements which belong to both A and B. It is denoted by '$A \cap B$'.
  $A \cap B = \{x : x \in A \text{ and } x \in B\}$

  Example: If $A = \{a, 1, x, p\}$ and $B = \{p, q, 2, x\}$, then $A \cap B = \{p, x\}$ since the elements p and x are in both A and B.

  If $A \cap B = \phi$, then A and B are called disjoint sets.
- Properties of operation of intersection:
  - $A \cap B = B \cap A$ (Commutative Law)
  - $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative Law)
  - $\phi \cap A = \phi$ (Law of ϕ)
  - $U \cap A = A$ (Law of U)
  - $A \cap A = A$ (Idempotent Law)
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law of $\cap$ on $\cup$)
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. This is called the distributive law of $\cap$ on$\cup$.
- The difference between the sets A and B (i.e., $A - B$, in this order) is the set of the elements which belong to A, but not to B.

  $A - B = \{x: x \in A \text{ and } x \notin B\}$

  Example: If $A = \{1, 4, 9, 16, 25, 36\}$ and $B = \{3, 9, 18, 36\}$, then $A - B = \{1, 4, 16, 25\}$ since the elements 1, 4, 16 and 25 are in A, but not in B.
- Some formulas of operation of difference:
  - $A - B = A - (A \cap B)$
  - For $A \neq B$, $A - B \neq B - A$
  - For $A = B$, $A - B = B - A = \phi$
  - For $B = \phi$, $A - B = A$ and $B - A = \phi$
  - $A - U = \phi$
  - For $A \subset B$, $A - B = \phi$; for this reason, $A - U = \phi$
- The sets $A - B$, $A \cap B$ and $B - A$ are mutually disjoint sets, i.e., the intersection of any of these two sets is a null set.



- If U is a universal set and A is a subset of U, then the complements of A are denoted by the set A'. This is the set of all element of U which are not the elements of A.

  $A' = \{x : x \in U \text{ and } x \notin A\} = U - A$

  A' is also the subset of U.

  Example: If $A = \{1, 3, 5\}$ and $U = \{1, 2, 3, 4, 5, 7, 9\}$, then $A' = U - A = \{1, 2, 3, 4, 5, 7, 9\} - \{1, 3, 5\} = \{2, 4, 7, 9\}$
- For any set A, $(A')' = A$
- For any sets A and B,

  $(A \cup B)' = A' \cap B'$

  $(A \cap B)' = A' \cup B'$

  These are called De Morgan's laws.
- Some more properties of the complement of a set:
  - $A \cup A' = U$
  - $A \cap A' = \phi$
  - $\phi' = U$ and $U' = \phi$
- If A and B are finite sets, such that $A \cap B = \phi$, then

  $n(A \cup B) = n(A) + n(B)$
- If A and B are finite sets, such that $A \cap B \neq \phi$, then,

  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Example: In a group of 40 girls, 30 like singing and 21 like dancing. If each girl likes at least one of the two (i.e., singing or dancing), then find

(1) the number of girls who like both singing and dancing

(2) the number of girls who like only singing or dancing

Solution:

Let X be the set of girls who like singing, and Y be the set of girls who like dancing.

Then, $X \cup Y$ is the set of girls who like at least one of the two (i.e., singing or dancing), and $X \cap Y$ is the set of girls who like both singing and dancing.

(1)

It is given that $n(X) = 30$, $n(Y) = 21$, $n(X \cup Y) = 40$

Using the formula, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$, we obtain

$40 = 30 + 21 - n(X \cap Y)$

$\Rightarrow n(X \cap Y) = 51 - 40 = 11$

Thus, 11 girls like both singing and dancing.

(2)

$X \cap Y'$ is the set of girls who like only singing

$X' \cap Y$ is the set of girls who like only dancing

$$n(X \cap Y') = n(X) - n(X \cap Y)$$
$$= 30 - 11$$
$$= 19$$
$$n(X' \cap Y) = n(Y) - n(X \cap Y)$$
$$= 21 - 11$$
$$= 10$$
$$n(X \cap Y') + n(X' \cap Y) = 19 + 10 = 29$$

The number of girls who like only singing or dancing =

$n(X \cap Y') + n(X' \cap Y) = 29.$

- ❖ If A, B and C are finite sets, then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$